

Finite Math - Fall 2018  
Lecture Notes - 9/11/2018

## HOMEWORK

- Section 3.1 - 9, 11, 15, 16, 18, 20, 22, 24, 26, 29, 34, 39, 50, 55, 58, 63, 71

### SECTION 3.1 - SIMPLE INTEREST

Suppose you make a deposit or investment of  $P$  dollars or you take out a loan of  $P$  dollars. The amount  $P$  is called the *principal*.

All of these things have an *interest rate* attached to them, essentially rent on the money, which is paid as *interest*.

**Simple Interest.** Simple interest is computed as

$$I = Prt$$

where  $I$  = interest,  $P$  = principal,  $r$  = annual simple interest rate (written as a decimal), and  $t$  = time in years.

**Example 1.** Suppose you deposit \$2,000 into a savings account with an annual simple interest rate of 6%. How much interest will accrue after 6 months?

**Solution.** 6 months is 0.5 years, so  $t = 0.5$ . The interest is 6%, so  $r = 0.06$ . The principal is  $P = 2000$ . Plug all this in to get

$$I = 2000(0.06)(0.5) = 60.$$

So, \$60 would have accrued after 6 months.

**Future Value.** Often, we might be more curious about how much will be in the account or how much will be owed on the loan after a certain period. This amount is called the *future value*. Another name for principal is *present value*. It is found by simply adding the original investment/loan amount to the interest accrued.

**Definition 1** (Future Value).

$$A = P + I = P + Prt$$

and in a simplified form

$$A = P(1 + rt)$$

where  $A$  = future value,  $P$  = principal/present value,  $r$  = annual simple interest rate,  $t$  = time in years.

**Example 2.** Suppose you take out a \$10,000 loan at a simple annual interest rate of 3.2%. How much would be due on the loan after 10 months?

**Solution.** *Principal*  $P = 10000$

*interest rate*  $r = 0.032$

*10 months*  $= \frac{10}{12}$  *years*  $= \frac{5}{6}$  *years, so*  $t = \frac{5}{6}$  *The future value is then*

$$\begin{aligned} A &= 10000 \left( 1 + (0.032) \left( \frac{5}{6} \right) \right) \\ &\approx 10000(1.027) = \$10,266.67 \end{aligned}$$

**Example 3.** *You make an investment of \$3,000 at an annual rate of 4.5%. What will be the value of your investment after 30 days? (Assume there are 360 days in a year.)*

**Solution.** *\$3,011.25*

We can also use the formulas to predict what interest rate we need or how much principal to take out/deposit.

**Example 4.** *You're looking to invest \$5,000 and make \$100 in interest after 10 weeks. What annual rate on your investment will you need to accomplish this?*

**Solution.**  *$I = 100$  and the time is  $t = \frac{10}{52}$  since there is 52 weeks in a year, so plugging all this into the formula gives*

$$100 = 5000r \left( \frac{10}{52} \right)$$

*and we need to solve for  $r$ .*

$$\begin{aligned} \implies 100 &= 5000r \left( \frac{10}{52} \right) \\ \implies r &= \frac{100}{5000 \left( \frac{10}{52} \right)} \\ &= 0.104 \end{aligned}$$

*So we would need an annual rate of 10.4% to make \$100 in interest after 10 weeks.*

**Example 5.** *You invest \$4,000 at an annual rate of 3.9%. How long will it take for the investment to be worth \$5,000? Give your answer in years, correct to 2 decimal places.*

**Solution.** *6.41 years*

One often uses a brokerage firm when making investments, many of which charge you a fee based on the transaction amount (principle) when both buying AND selling stocks.

**Example 6.** Suppose a brokerage firm uses the following commission schedule

<i>Principal</i>	<i>Commission</i>
Under \$3,000	\$25+1.8% of principal
\$3,000 - \$10,000	\$37+1.4% of principal
Over \$10,000	\$107+0.7% of principal

An investor purchases 450 shares of a stock at \$21.40 per share, keeps the stock for 26 weeks, then sells the stock for \$24.60 per share. What was the annual interest rate earned on the investment?

**Solution.** To purchase 450 shares will cost  $\$21.40(450) = \$9,630$ . This falls into the second fee range of the commission schedule, so the transaction fee will be

$$\$37 + 0.014(\$9630) = \$171.82.$$

Thus, the total initial investment is

$$\$9,630 + \$171.82 = \$9,801.82.$$

Next, the investor sells the stock for

$$\$24.60(450) = \$11,070$$

This falls into the third fee range on the schedule, so the commission is

$$\$107 + 0.007(\$11,070) = \$184.49.$$

Thus, the net return on the investment is

$$\$11,070 - \$184.49 = \$10,885.51.$$

Now, using the total investment as the principal and the net return as the future value, we can use the future value formula to figure out the annual interest rate earned.  $P = 9801.82$ ,  $A = 10885.51$ , the time elapsed was 26 weeks, and there are 52 weeks in a year, so  $t = \frac{26}{52} = 0.5$ .

$$\begin{aligned} 10885.51 &= 9801.82(1 + 0.5r) \\ &= 9801.82 + 4900.91r \\ \implies 1083.69 &= 4900.91r \\ \implies r &= \frac{1083.69}{4900.91} \approx 0.22112 \end{aligned}$$

So the interest rate earned was 22.112%.